STRUCTURE THEORY OF d-ALGEBRAS

In many branches of higher mathematics, the concept of an algebra is fundamental. Roughly speaking, an algebra is a set of elements together with three properties:

- Any two elements can be added or subtracted.
- Any two elements can be multiplied.
- Any element can be "scaled" by a scalar.

A classic example of an algebra that is used throughout mathematics is the set of polynomials with real number coefficients. That is, anything of the form $p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, where all $a_1, \ldots, a_n$ are real numbers. We can add, subtract, and multiply two polynomials in the usual way, and we can scale a polynomial by multiplying it by a real number.

The polynomial algebra happens to have the property that mutliplication is *commutative*, or in other words, $p(x)q(x) = q(x)p(x)$ for all polynomials $p(x)$ and $q(x)$. Algebras which have this property are called *commutative algebras*. Since commutative algebras play such a significant role in higher mathematics, they have been very thouroughly studied, and there is a wealth of results and theorems out there that describe the properties and features of commutative algebras.

This paper studies objects called d-algebras, which are essentailly algebras which are "almost" commutative. What this means is that instead of $ab = ba$ (regular commutativity), we have $ab = ba + d(a)d(b)$ (d-algebra "commutativity"), where d is some function defined on our algebra. In this sense, the $d(a)d(b)$ is an error term getting in the way of commutativity. As it turns out, d-algebras are useful for Supergeometry and Supersymmetry, which are concepts in modern theoretical physics that describe the nature of elmentary particles.

For this reason, this paper studies the structure of d-algebras. Most importantly, it generalizes a few of the important theorems about commutative algebras to these "almost commutative" algebras, making them easier to work with.